The Causal Interpretation of Dust and Radiation Fluids Non-Singular Quantum Cosmologies

J. Acacio de Barros[*a]
N. Pinto-Neto[†b]
and
M. A. Sagioro-Leal [a]
[a]*Departamento de Física - ICE*
Universidade Federal de Juiz de Fora
36036-330, Juiz de Fora, MG, Brazil
[b]*Centro Brasileiro de Pesquisas Físicas/Lafex*
Rua Xavier Sigaud, 150 - Urca
22290-180, Rio de Janeiro, RJ,Brazil

December 31, 2013

Abstract

We apply the causal interpretation of quantum mechanics to homogeneous and isotropic quantum cosmology where the sources of the gravitational field are either dust or radiation perfect fluids. We find non-singular quantum trajectories which tends to the classical one when the scale factor becomes much larger then the Planck length. In this situation, the quantum potential becomes negligible. There are no particle horizons. As radiation is a good approximation for the matter content of the early universe, this result suggests that the universe can be eternal due to quantum effects.

PACS number(s): 98.80.H, 03.65.Bz

[*]e-mail address: acacio@fisica.ufjf.br
[†]e-mail address: nen@lca1.drp.cbpf.br

1 Introduction

The appearance of initial singularities in the classical cosmological models which better describe the universe we live in constitutes a big puzzle to all cosmologists. Until now, singularities are out of the scope of any physical theory. If we assume that a physical theory can describe the whole Universe at every instant, even at its possible moment of creation (which is the best attitude because it is the only way to seek the limits of physical science), then these classical singular points must be avoided. Indeed, no one expects that classical general relativity continues to be valid under extreme situations of very high energy density and curvature. In particular, it is very plausible that quantum gravitational effects become important under these conditions, eliminating the singularities that appear classically. To see if this is indeed the case, we should construct a theory of quantum cosmology. However, any quantum theory when applied to cosmology presents new profound conceptual problems. How can we apply the standard probabilistic Copenhaguen interpretation to a single system as the Universe? Where in a quantum Universe can we find a classical domain where we could construct our classical measuring apparatus to test and give sense to the quantum theory? Who are the observers of the whole Universe? This is not a problem of quantum gravity alone, because there is no problem with the concept of an ensemble of black holes and a classical domain outside it, but it is specific of quantum cosmology. As we cannot apply the Copenhaguen interpretation to quantum cosmology, we will adopt an alternative non-probabilistic interpretation, which circumvents the measurement problem because it is an ontological interpretation of quantum mechanics: it is not necessary to have a measuring apparatus or a classical domain in order to recover physical reality. It is the causal interpretation of quantum mechanics [1, 2].

In this letter, we apply the causal interpretation of quantum mechanics to some specific quantum states already found in the literature [3, 4], and show that the Bohmian (quantum) trajectories are non-singular, tending to the classical trajectories when the scale factor is large. The quantum potential is the responsible for this behavior. When the scale factor is small, the quantum potential becomes large creating an effective repulsive force avoiding the singularity. When the scale factor is large, the quantum potential becomes negligible and the classical potential dominates. The minisuperspace models we study are constituted of a Friedman-Robertson-Walker (FRW) metric with either dust or radiation perfect fluids [3, 4]. The ADM quantization procedure is performed because in these cases there is a preferable time variable which rends the quantum equations into a very simple Shroedinger form. The quantum solutions obtained are gaussians [3, 4]. The dust case is rather academic but the radiation case is important because it can mimic quite well the matter content at the very early universe. In all cases we obtain non-singular eternal models without any particle horizon.

This letter is organized as follows. In the next section we make a summary of the causal interpretation and its application to quantum cosmology. In section 3 we abridge the results of Refs. [3, 4] which concern this letter. In section 4 we apply the causal interpretation to the solutions presented in section 3, obtaining the Bohmian trajectories. We compare our results with those of Refs. [3, 4]. We end with comments and conclusions.

2 The causal interpretation of quantum mechanics

In this section, we will review the causal interpretation of quantum mechanics. Let us begin with the Schrödinger equation, in the coordinate representation, for a non-relativistic particle with the hamiltonian $H = p^2/2m + V(x)$:

$$i\hbar \frac{d\Psi(x,t)}{dt} = [-\frac{\hbar^2}{2m}\nabla^2 + V(x)]\Psi(x,t). \tag{1}$$

Writing $\Psi = R\exp(iS/\hbar)$, and substituting it into (1), we obtain the following equations:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = 0, \tag{2}$$

$$\frac{\partial R^2}{\partial t} + \nabla.(R^2 \frac{\nabla S}{m}) = 0. \tag{3}$$

The usual probabilistic interpretation takes equation (3) and understands it as a continuity equation for the probability density R^2 for finding the particle at position x and time t if a measurement is performed.

All physical information about the system is contained in R^2, and the total phase S of the wave function is completely irrelevant. In this interpretation, nothing is said about S and its evolution equation (2). However, examining equation (3), we can see that $\nabla S/m$ may be interpreted as a velocity field, suggesting the identification $p = \nabla S$. Hence, we can look to equation (2) as a Hamilton-Jacobi equation for the particle with the extra potential term $-\hbar^2 \nabla^2 R/2mR$.

After this preliminary, let us introduce the causal interpretation of quantum mechanics, which is based on the *two* equations (2) and (3), and not only in the last one as is the Copenhaguen interpretation:

i) A quantum system is composed of a particle *and* a field Ψ (obeying the Schrödinger equation (1)), each one having its own physical reality.

ii) The quantum particles follow trajectories $x(t)$, *independent on observations*. Hence, in this interpretation, we can talk about trajectories of quantum particles, contrary to the Copenhaguen interpretation, where only positions at one instant of time have a physical meaning.

iii) The momentum of the particle is $p = \nabla S$.

iv) For a statistical ensemble of particles in the same quantum field Ψ the probability density is $P = R^2$. Equation (3) guarantees the conservation of P.

Let us make some comments:

a) Equation (2) can now be interpreted as a Hamilton-Jacobi type equation for a particle submited to an external potential which is the classical potential plus a new quantum potential

$$Q \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}. \tag{4}$$

Hence, the particle trajectory $x(t)$ satisfies the equation of motion

$$m\frac{d^2 x}{dt^2} = -\nabla V - \nabla Q. \tag{5}$$

b) Even in the regions where Ψ is very small, the quantum potential can be very high, as we can see from equation (4). It depends only on the form of Ψ, not on its absolute value. This fact brings home the non-local and contextual character of the quantum potential[1]. This is very important because Bell's inequalities together with Aspect's experiments show that, in general, a quantum theory must be either non-local or non-ontological. As Bohm's interpretation is ontological, it must be non-local, as it is. The quantum potential is responsible for the quantum effects.

c) This interpretation can be applied to a single particle. In this case, equation (3) is just an equation to determine the function R, which forms the quantum potential acting on the particle via equation (5). It is not necessary to interpret R^2 as a probability density and hence it may not be normalizable. The interpretation of R^2 as a probability density is appropriate only in the case mentioned in item (iv) above. The causal interpretation is not, in essence, a probabilistic interpretation.

d) The classical limit is very simple: we only have to find the conditions for having $Q = 0$.

e) There is no need to have a classical domain because this interpretation is ontological. The question on why in a real measurement we do not see superpositions of the pointer apparatus is answered by noting that, in a measurement, the wave function is a superposition of non-overlaping wave functions. The particle will enter in one region, and it will be influenced by the unique quantum potential obtained from the sole non-zero wave function defined on this region. The particle cannot jump to other branchs because it cannot pass through nodal points of the wave function.

In the next section we will perform an ADM quantization of Friedman-Robertson-Walker minisuperspace models containing either dust or radiation perfect fluids. In these cases, a prefered time variable can be selected in terms of potentials of the velocity field of the fluids. As a time variable will be chosen before quantization, the quantum equations will be exactly like the Schrödinger equation. Instead of the particle position, as in the above example, the single degree of freedom of these quantum models will be the scale factor of the universe. The interpretation of the quantum solutions will run analogously to what was described above. The time evolution of the scale factor will be different from the classical one due to the presence of an extra quantum potential term in the modified Hamilton-Jacobi equation it satisfies.

[1] This fact becomes evident when we generalize the causal interpretation to a many particles system.

3 The ADM quantization of dust and radiation minisuperspace models

In this section we present the minisuperspace models which we will analyze using the causal interpretation. These models are obtained from the quantization of the dust and radiation fluids through the ADM prescription.

For the quantization of the radiation filled FRW model we follow Ref. [4]. We start with the line element

$$ds^2 = -N(t)^2 dt^2 + a^2(t)\sigma_{ij} dx^i dx^j .$$

where σ_{ij} is the metric of constant curvature three-surfaces. The full action for a perfect fluid is given by

$$S = \int_M d^4x \sqrt{-g}(R^{(4)} + p) + 2\int_{\partial M} d^3x \sqrt{h} h_{ij} K^{ij}, \tag{6}$$

where p is the pressure, $R^{(4)}$ is the scalar curvature, h_{ij} is the 3-metric on ∂M, K^{ij} is the second fundamental form of the boundary, and we choose $c = 16\pi G = 1$.

The action (6) can be reduced to

$$S_r = \int dt \left[\dot{a}\, p_a - \dot{\varphi} p_\varphi + \dot{S} p_S - N\mathcal{H}\right],$$

where we are using Schutz's fluid variables φ, λ, γ, θ, S [5], in terms of which the four-velocity of the fluid is written as

$$U_\nu = \frac{1}{\mu}(\partial_\nu \varphi + \lambda \partial_\nu \gamma + \theta \partial_\nu S). \tag{7}$$

The quantity μ is the specific enthalpy. The super-Hamiltonian $\mathcal{H}$ takes the form

$$\mathcal{H} = -\left(\frac{p_a^2}{24a} + 6ka\right) + p_\varphi^{4/3} a^{-3} e^S .$$

For details, see Refs. [6, 7].

Performing an ADM reduction using the conformal-time gauge $N = R$ in the radiation case, the reduced action becomes [7]

$$S_r = \int dt \left[\dot{a}\, p_a - \left(\frac{p_a^2}{24} + 6ka^2\right)\right],$$

where $k = +1$, 0 or -1, for spherical, flat, or hyperbolic spacelike sections of the three space with metric σ_{ij}, respectively. The Hamiltonian in the reduced phase space, which has only a as a degree of freedom, takes the very simple form

$$H = \frac{p_a^2}{24} + 6ka^2 .$$

The classical solutions are:

$$a = a_0 \begin{cases} \sin t, & \text{for } k = 1 \\ t, & \text{for } k = 0 \\ \sinh t, & \text{for } k = -1 \end{cases} . \tag{8}$$

which are the well known solutions for radiation in conformal time.

The quantized Hamiltonian, in units where $\hbar = 1$, is

$$\hat{H} = -\frac{1}{24}\frac{d^2}{da^2} + 6ka^2 . \tag{9}$$

As $a \geq 0$, the requirement that $\hat{H}$ must be self-adjoint leads to the restriction

$$\psi'(0) = \alpha\psi(0) \tag{10}$$

on the wave function ψ, where α is a parameter in the interval $(-\infty, \infty]$. Two solutions will be obtained, one for $\alpha = 0$ and the other for $\alpha = \infty$.

The propagators for the Hamiltonian (9) are well known. However, we have the extra constraint that $a \geq 0$. For this reason, the Hilbert space is restricted to functions in $L^2(0,\infty)$. For the case where $\alpha = 0$ the propagator is

$$G^{(I)}(a,a',t) = G(a,a',t) + G(a,-a',t), \tag{11}$$

where $G(a,a',t)$ is the usual harmonic oscillator propagator for a system with mass $m = 12$ and angular frequency $\omega = \sqrt{k}$. Seting $\alpha = 0$ in Eq. (10), we take a gaussian wavepacket as the initial normalized state,

$$\psi_0^{(I)}(a) = \left(\frac{8b}{\pi}\right)^{1/4} \exp(-\beta a^2), \tag{12}$$

where $\beta = b + iB$, B and b are real, and $b > 0$. The wave function at time t given by Eqs. (11) and (12) is

$$\begin{aligned} \psi^{(I)}(a,t) &= \left(\frac{8b}{\pi}\right)^{1/4} \left[\frac{6\sqrt{k}}{\cos(\sqrt{k}t)[\beta\tan(\sqrt{k}t) - 6i\sqrt{k}]}\right]^{1/2} \\ &\times \exp\left\{\frac{6i\sqrt{k}}{\tan(\sqrt{k}t)}\left(1 + \frac{6i\sqrt{k}}{\cos^2(\sqrt{k}t)[\beta\tan(\sqrt{k}t) - 6i\sqrt{k}]}\right) a^2\right\}. \end{aligned} \tag{13}$$

The expectation value for the scale factor a can be computed from (13), and is

$$\langle\hat{a}\rangle_t^{(I)} = \frac{1}{12}\sqrt{\frac{2}{\pi b}} \left\{ \begin{array}{ll} \sqrt{b^2\sin^2 t + (6 - B\tan t)^2\cos^2 t} & \text{for } k = 1 \\ \sqrt{b^2 t^2 + (6 - Bt)^2} & \text{for } k = 0 \\ \sqrt{b^2\sinh^2 t + (6 - B\tanh t)^2\cosh^2 t} & \text{for } k = -1 \end{array} \right. . \tag{14}$$

For $\alpha = \infty$ the propagator is

$$G^{(II)}(a,a',t) = G(a,a',t) - G(a,-a',t). \tag{15}$$

We take as the initial state the wave packet

$$\psi_0^{(II)}(R) = \left(\frac{8b}{\pi}\right)^{1/4} R\exp(-\beta R^2). \tag{16}$$

The evolution of (16) governed by the propagator (15) is

$$\begin{aligned} \psi^{(II)}(a,t) &= \left(\frac{128b^3}{\pi}\right)^{1/4} \left[\frac{216ik^{3/2}}{\sin^3(\sqrt{k}t)}\right] \left[\beta - \frac{6i\sqrt{k}}{\tan(\sqrt{k}t)}\right]^{-3/2} \\ &\times R\exp\left\{\frac{6i\sqrt{k}}{\tan(\sqrt{k}t)}\left(1 + \frac{6i\sqrt{k}}{\cos^2(\sqrt{k}t)[\beta\tan(\sqrt{k}t) - 6i\sqrt{k}]}\right) R^2\right\}. \end{aligned} \tag{17}$$

The expectation value for the scale factor a with the wavefunction (17) is

$$\langle\hat{a}\rangle_t^{(II)} = 2\langle\hat{a}\rangle_t^{(I)}. \tag{18}$$

We now turn our attention to the dust filled minisuperspace model presented by Gotay and Demaret [3]. Once again Schutz's variables and a FRW metric are used. In this case the field φ in Eq. (7) is the only velocity potential which is non null. The time to be chosen will be the proper time of the dust particles defined by $\varphi = -t$, which is equivalent to choose $N = 1$. Seting $p = 0$ and $\mu = 1$, the ADM reduced super-Hamiltonian becomes (see Ref. [3])

$$\mathcal{H} = -\left(\frac{p_a^2}{24a} + 6ka\right) - p_\varphi.$$

The reduced Hamiltonian, with the above choice of time, is

$$H(a,p_a) = \frac{p_a^2}{24a} + 6ka.$$

If we perform the canonical transformation

$$x = \frac{4}{3}\sqrt{6}a^{3/2}, \qquad p_x = \frac{\sqrt{6}}{12}a^{-1/2}p_a$$

the Hamiltonian takes the simple form

$$H(x, p_x) = p_x^2 + Kx^{2/3},$$

where $K = \frac{3}{2}6^{3/2}k$. Taking $k = 0$, the classical solution is simple to obtain. It is $x(t) = t$ or equivalently $a(t) \propto t^{2/3}$. The quantized Hamiltonian for $k = 0$ is, in units where $\hbar = 1$,

$$\hat{H} = -\frac{d^2}{dx^2}.$$

Once again, the requirement of self-adjointness of $\hat{H}$ yields the boundary conditions $\psi'(0) = \alpha\psi(0)$. Choosing $\alpha = 0$, we can evolve the initial gaussian wave function

$$\psi_0(x) = \left[\frac{8b}{\pi}\right]^{1/4} \exp(-\beta x^2), \tag{19}$$

by applying the corresponding propagator, analogously to what was done in the radiation case. As before, $\beta = b + iB$ with $b \geq 0$. The wave function at time t is given by:

$$\psi(x, t) = \left[\frac{8b}{\pi}\right]^{1/4} (1 + 4i\beta t)^{-1/2} \exp\left[\frac{-\beta x^2}{1 + 4i\beta t}\right]. \tag{20}$$

The expectation value for x can easily be computed, and is

$$\langle \hat{x} \rangle_t = \left(\frac{1}{2\pi b}\right)^{1/2} \left[16b^2t^2 + (1 - 4Bt)^2\right]^{1/2}. \tag{21}$$

We will now turn our attention to the causal interpretation of the wavefunctions $\psi^{(I)}$, $\psi^{(II)}$, and ψ.

4 The causal interpretation

The causal interpretation of the above minisuperspace models is straightforward. Let us begin by the dust field. In this case, the quantum trajectories are solutions of the following differential equation:

$$p_x = \frac{1}{2}\dot{x} = \frac{\partial S_d}{\partial x}, \tag{22}$$

where S_d is the phase of the wave function (20). The general solution is:

$$x(t) = x_0 \; [16b^2t^2 + (1 - 4Bt)^2]^{1/2}, \tag{23}$$

where x_0 is an arbitrary positive integration constant. That the mean value of x founded in the previous section is, apart from the integration constant, the same function of time as the solution (24) is not surprising. Mean values in the causal interpretation are the same as in the usual interpretation if we also assume that the amplitude squared of the wave function, $R^2(x, 0)$, represents a probability density distribution of initial conditions of the quantum trajectories. Then, we obtain from Eq. (19)

$$\begin{aligned} \langle x(t) \rangle &= \int_0^\infty \left(\frac{8b}{\pi}\right)^{1/2} e^{-2bx_0^2} x_0 [16b^2t^2 + (1 - 4Bt)^2]^{1/2} dx_0 \\ &= (2\pi b)^{-1/2} [16b^2t^2 + (1 - 4Bt)^2]^{1/2} \,, \end{aligned} \tag{24}$$

which is the result (21) of the previous section.

It can be seen from Eq. (23) that no quantum trajectory is singular. The scale factor is never zero for any x_0 greater than zero. Also, the trajectories approach the classical one for large $|t|$ (remember that $x(t) \propto a^{3/2}(t)$). This behavior can be seen by examining the quantum potential,

$$Q_d = -2b\frac{[2b(x^2 - 8bt^2) - (1 - 4Bt)^2]}{[(1 - 4Bt)^2 + 16b^2t^2]^2}. \tag{25}$$

Along the quantum trajectory (23), the quantum potential turns out to be:

$$Q_d(t) = -\frac{2bx_0^2(1 - 2bx_0^2)}{x^2(t)}, \tag{26}$$

where $x(t)$ is given by (23).

From the above equation we can see that the quantum potential goes to zero when $|t|$ becomes large but it is positive of order b when $|t|$ is small[2]. In this situation, it works like a repulsive force around the region $x = 0$. Figure 1 shows the curves $x(t)$ and $Q(t)$ (along the trajectory) versus t. They represent classical universes contracting from infinity to a minimum size, where its behavior is not classical, and then expanding to infinity, getting classical again as the scale factor becomes large. These models have no particle horizon. The integral $\int_{-\infty}^{t} a^{-1}(t')\mathrm{d}t' \propto \int_{-\infty}^{t} x^{-2/3}(t')\mathrm{d}t'$ diverges.

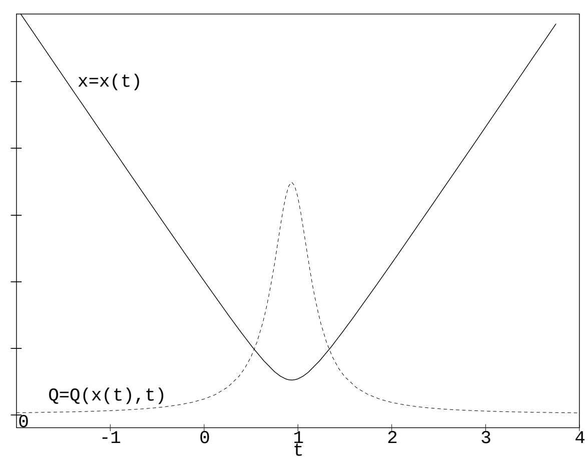

Fig. 1: A typical trajectory for $x(t)$ and its corresponding quantum potential (dashed line). We see that when x approaches the classical singularity the quantum potential becomes large.

Let us now turn our attention to the more realistic case of radiation. Again, the application of the causal interpretation is straightforward. We have to take the phases of the wave functions (13,17) for the cases $k = 0, k \pm 1$, calculate their derivatives with respect to a, and equate the result with $p_a = 12\dot{a}$. The solutions of these first order differential equations are:

$$a(t) = a_0 \begin{cases} \sqrt{b^2 \sin^2 t + (6 - B\tan t)^2 \cos^2 t} & \text{for } k = 1 \\ \sqrt{b^2t^2 + (6 - Bt)^2} & \text{for } k = 0 \\ \sqrt{b^2 \sinh^2 t + (6 - B\tanh t)^2 \cosh^2 t} & \text{for } k = -1 \end{cases} . \tag{27}$$

[2]There is nothing special with the choice $x_0 = 1/\sqrt{2b}$. The quantum potential is zero along the trajectory but the quantum force $F_d(t) = -\partial_x Q_d$ is not.

where a_0 is a positive integration constant. Like before, they have the same functional behaviour as the mean values encountered in the previous section for the reasons already explained in the dust case. None of these solutions reach the singular point $a = 0$ .The quantum potentials are given by

$$Q = 3b \begin{cases} \frac{[b(b\sin^2(t)-72a^2)+(6-B\tan(t))^2\cos^2(t)]}{[(6-B\tan(t))^2\cos^2(t)+b^2\sin^2(t)]^2} & \text{for } k=1 \\ \frac{[b(bt^2-72a^2)+(6-Bt)^2]}{[(6-Bt)^2+b^2t^2]^2} & \text{for } k=0 \\ \frac{[b(b\sinh^2(t)-72a^2)+(6-B\tanh(t))^2\cosh^2(t)]}{[(6-B\tanh(t))^2\cosh^2(t)+b^2\sinh^2(t)]^2} & \text{for } k=-1 \end{cases} . \tag{28}$$

At the trajectories, the quantum potentials for $k = 0, 1, -1$ are the same, and equal to

$$Q = \frac{3ba_0^2(1-72ba_0^2)}{a^2(t)}, \tag{29}$$

where $a(t)$ is given by (27).

For $k = 0$ and $k = -1$ the models are qualitatively similar to the dust case. It can be seen from Eq. (27) that these universes contract classically from infinity to a minimum size, where their behaviors are not classical, and then expand to infinity, getting classical again as the scale factor becomes large. The case $k = 1$ deserves special attention. Examining the case with $B = 0$ and $b > 6$ (the general case is qualitatively similar), we can see that the quantum trajectory oscillates between the minimum value $6a_0$ and the maximum value $a_0 b$. Their ratio is $b/6$. The ratio between the quantum and classical forces at these points are $-b^2/36$ for the minimum and $-36/b^2$ for the maximum. The universe we live in is large and classical, and it must have gone through a contracted phase where nucleosynthesis took place. This means that the ratio between the maximum and minimum values of the radius of the universe (which is the inverse ratio of the respective temperatures at these epochs since the fluid is radiation) must be at least of the order of 10^{10}. Hence b must be greater than 10^{10}, yielding a very flat initial gaussian wave function. This ensures that the universe had a nucleosynthesis era with a classical behaviour since then.

These models do not have particle horizons. In this case the particle horizon is proportional to the integral $\int_{-\infty}^{t} a^{-1}(t')N(t')\mathrm{d}t'$ which in the gauge $N = a$ we are using evidently diverges.

Figures 2 and 3 show the scale factor and the quantum potential for $k = 0$ and $k = -1$. When a is large, the quantum potentials go to zero, while when a approaches its minimum size, they become important, creating an effective repulsive quantum force around this region. Figures 4 and 5 show the scale factor and the quantum potential for $k = 1$ and for different values of b. Note that for larger b's the amplitude of oscillation also becomes larger.

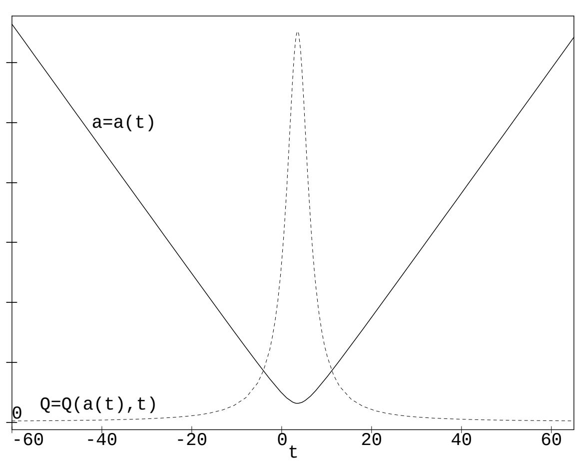

Fig. 2: Typical quantum trajectory with its corresponding quantum potential (in dashed lines) for the radiation fluid model for the case where $k = 0$.

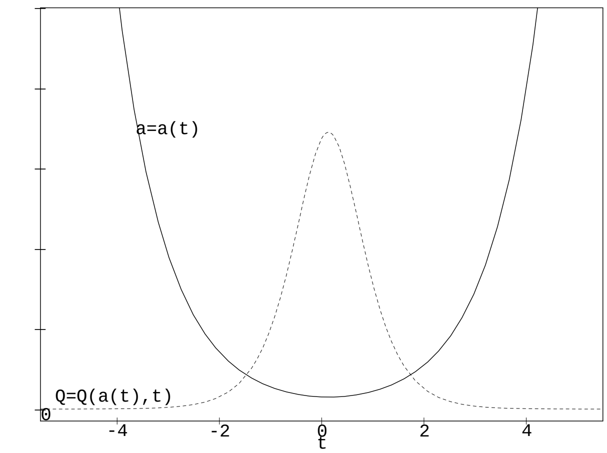

Fig. 3: Typical quantum trajectory with its corresponding quantum potential (in dashed lines) for the radiation fluid model for the case where $k = -1$.

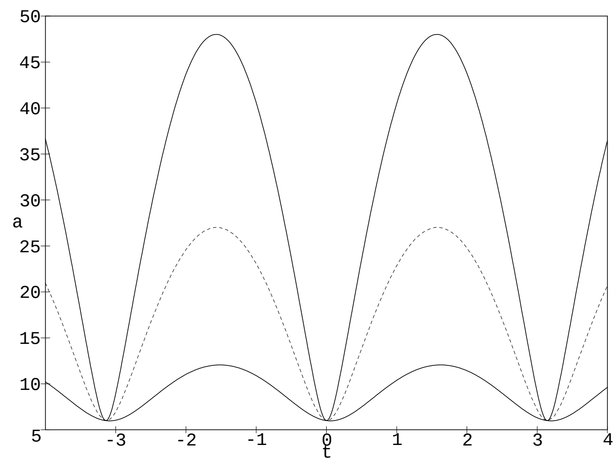

Fig. 4: Quantum trajectories for different values of b and the same initial condition $a(0)$ for the radiation fluid model and $k = 1$. The larger the value of b the larger is the amplitude of oscillation.

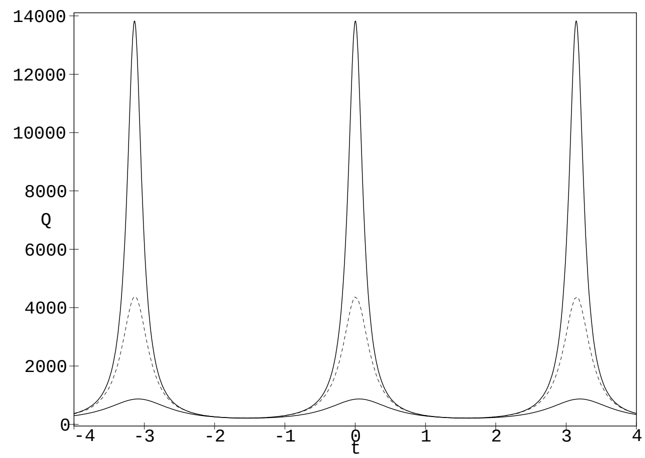

Fig. 5: Quantum potentials computed along the quantum trajectories shown in Fig. 4. The dashed quantum potential corresponds to the dashed trajectory in Fig. 4. We can see that the value of the quantum potential increases as we approach a classical singularity, being higher when b is higher.

5 Conclusion

In this letter we have obtained a class of non-singular cosmological models without particle horizons by applying the causal interpretation to some quantum states of the universe already obtained in the literature [3, 4] and presented in section 3. We have shown that the Bohmian trajectories are the same functions of time as their corresponding mean-values obtained in Refs. [3, 4], which makes use of the conventional Copenhaguen interpretation. However, it seems to us that only in the causal approach we can arrive at definite conclusions about the existence of singularities. It is not because the mean value of a variable is different from zero at all times that this variable cannot be zero sometime. Nothing forbids that this be the case for the single universe we live in. The causal interpretation applied to this problem states that each individual trajectory of these quantum states is not singular, a much more stronger and valuable result. This means that if the universe is in one of these quantum states, then it is indeed non-singular because there is no quantum trajectory with singularities.

For the more realistic radiation fluid, we have obtained singularity-free models, without particle horizons, which approach the classical behavior as the universe expands. For flat and negative curvature three-surfaces this is the situation in all cases. The universe contracts classically from infinity to a minimum size, where its behavior is not classical, and then expands to infinity, getting classical again as the scale factor becomes large. For the positive curvature case, the classical behavior is achieved only for some values of the parameters. In this case, we would have an eternal periodic universe which is classical when it is large.

These examples show that quantum gravitational effects can indeed prevent the formation of cosmological singularities but this result can only be stated with strength along the lines of the causal interpretation. It should be interesting to investigate if the results presented here are stable under small perturbations. Then we will have to face new technical and interpretational problems. But this is another story.

Acknowledgments

Part of this work was done while NPN was a PREVI (Special Program for Visiting Professor/Researcher) fellow at the Physics Department of the Federal University at Juiz de Fora. We would like to thank the group of the “Pequeno Seminário” at CBPF for useful discussions and FAPEMIG for financial support. NPN would like to thank CNPq for financial support and the Federal University at Juiz de Fora, UFJF, for hospitality. JAB would like to thank the Laboratory for Cosmology and Experimental High Energy Physics (Lafex) at the Brazilian Center for Physical Research (CBPF/CNPq) for hospitality. MASL is supported by a scholarship from Capes.

References

[1] D. Bohm and B. J. Hiley, *The undivided universe: an ontological interpretation of quantum theory* (Routledge, London, 1993).

[2] P. R. Holland, *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Interpretation of Quantum Mechanics* (Cambridge University Press, Cambridge, 1993).

[3] M. J. Gotay and J. Demaret, Phys. Rev. **D28**, 2402 (1983).

[4] N. A. Lemos, J. Math. Phys. **37**, 1449 (1996).

[5] B. F. Schutz, Phys. Rev. **D2**, 2762 (1970); **D4**, 3559 (1971).

[6] V. G. Lapchinskii and V. A. Rubakov, Theo. Math. Phys. **33**, 1076 (1977).

[7] F. J. Tipler, Phys. Rep. **137**, 231 (1986).